Math curriculum Secondary 2 Scientific

ALGEBRA (44 h)

1. BASICS (6 h)

The student already knows how to manipulate sets and elementary operations on sets (union, intersection, etc...). He has dealt with ordered pairs and the Cartesian product of two sets. The aim of this theme is to study binary relations that play an important role in mathematics, in particular at the level of systematization and unification of ideas.

CONTENTS	OBJECTIVES	COMMENTS
1.1. Binary relations.	1. Identify a binary relation on a set. 2. List the elements of the graph of a binary relation on a finite set. 3. Identify an equivalence relation. 4. List the members of the equivalence class of an element. 5. Determine the partition associated with an equivalence relation. 6. Identify an order relation. • Identify a binary relation on a set. • List the elements of the graph of a binary relation on a finite set. • List the elements of the set associated with a given element by a binary relation on a finite set. • Recognize an equivalent relation. • List the members of the equivalence class of an element. • Determine the partition associated with an equivalence relation. • Construct the equivalence relation which determines a given partition.	The main objective is to introduce the notion of binary relation on a set and the graph of such a relation. The notions of reflexivity, symmetry, antisymmetry and transitivity will be introduced as we study equivalence and order relations. We will make sure to mention that the graph of a binary relation on a set E is a subset of the Cartesian product $E^2 = E \times E$. A binary relation on a set is denoted in general by a letter such as R, S, etc. The notation xRy means that the element x is related to the element y by the relation R. In general, the graph of the relation R is denoted by $G(R)$ or G_R. Theoretical development is to be avoided. We initiate the student to the notion of an equivalence relation through the definition and simple examples emphasizing partition which it determines on a given set, and the equivalence classes that it defines. It's recommended to neglect the notion of quotient set which is rather delicate to handle.
	• Recognize an order relation. • Recognize two non-comparable elements by an order relation.	When we have an equivalence relation R, the set $C(a) = \{x \in E \,/\, aRx\}$ is called "the equivalence class of a modulo R" and may be denoted simply by $\overline{a}$. Concerning the order relation and without studying order sets in details, we propose to show that an order is a mathematical structure which goes far beyond the context of real numbers. To this end, it is important to initiate the student to order relations other than the usual order on real numbers. In particular, he must become familiar with order relations where elements may not be comparable. We make sure to denote an arbitrary order relation by R and the usual order relation by $\leq$ on real numbers so that we can avoid every possible confusion.

2. NUMERICAL AND LITERAL CALCULATIONS (6 h)

The introductory study of arrangements and *p-lists*, that was accomplished in the first year of secondary education, makes it easier to study them more generally in the second year.

The student will have the chance to study arrangements with and without repetitions systematically, as well as general formulas that concern them. We underline the importance of this study in calculating probabilities.

CONTENTS	OBJECTIVES	COMMENTS
2.1. Arrangements and permutations.	1. Calculate $n!$. 2. Identify an arrangement, an arrangement with repetition, a permutation.	A_n^p denotes the number of arrangements without repetition, of p elements of a set of n elements $(p \leq n)$. The number of arrangement with repetition, or *p-lists*, of a set of n elements, is the cardinal number n^p of E^p.

CONTENTS	OBJECTIVES	COMMENTS
	3. Give and use formulas of the number of arrangements with repetition, of the number of arrangements without repetition, and of the number of permutations. • Calculate $n!$ where n is a natural number. • Recognize an arrangement without repetition of p elements of a set E of n elements. $(0 < p \le n)$. • Recognize a permutation. • Recognize an arrangement with repetition. • Know and use the formulas that give the number of arrangements with repetition, without repetitions, and number of permutations.	Direct calculation starting with the formulas and the use of a calculator are both recommended. We use the notation $n!$ (*n factorial*) and we especially define 0!. We make sure to choose examples from daily life as applications.

3. EQUATIONS AND INEQUALITIES (20 h)

Having mastered equations and inequalities of degree one as well as the technique of dividing a plane into regions, the student will consolidate his knowledge in this area, on the one hand by the manipulating problems of linear optimization and systems of three equations in three unknowns, and on the other hand, by studying polynomials of degree two in details.

Systems of linear inequalities have their natural field of application in optimization problems, mainly in economics. They allow one to find, under a certain number of constraints, conditions that favor a maximal benefit, a minimal loss and a better unfolding of a project.

Problems of degree two are at the base of most activities of calculations and graphical activities that will be met in the future.

CONTENTS	OBJECTIVES	COMMENTS
3.1. System of linear equations(3×3). Linear programming.	1. Translate the constraints of a linear programming problem into the form of a system of linear inequalities and an economic function. 2. Find graphically the optimal solution of a problem of linear programming. 3. Solve a system of linear equations (3×3). • Solve graphically a system of n linear inequalities $(2 \le n \le 5)$ in two unknowns. • Translate the constraints of a linear programming problem to a system of linear inequalities in two unknowns and give an optimal solution. • Solve a system of three linear equations in three unknown by the method of combinations and by the method of substitution. • Write in row-echelon form a system of three linear equations in three unknowns and solve it (Gauss method). • Recognize linear systems (3×3) that do not have solutions and those that have infinitely many solutions and write the solutions of these systems.	Linear systems of n inequalities in two unknowns could be dealt with at will for $n = 2$ or $n = 3$. If $n \ge 4$, they should be carefully chosen and should involve simple inequalities of the type $x \ge a$ or $y > a$. It will be interesting to include in these systems of inequalities equations of the type $ax+by = c$. The study of linear programming is done exclusively through examples. Every theoretical justification is to be avoided. Concerning the system of three equations in three unknowns, the method of combinations and the method of substitution furnish an important mathematical tool. However the student must take with care the first method because it sometimes leads to a result which does not satisfy all the equations. The method of Gauss becomes more interesting and advantageous as the number of equations increases. Systems leading to infinitely many solutions or no solution should be dealt with mainly through numerous examples. All indicative formulations especially the one involving determinants is not recommended. Every systematic study of parametric systems is to be avoided.
3.2. Polynomials, equations and inequalities of degree two.	1. Write a quadratic trinomial in its standard form. 2. Determine if a quadratic equation with real coefficients has real roots and find the number of these roots. 3. Calculate the real roots of a quadratic equation with real coefficients, if they exist.	The standard form of a polynomial of degree two $f(x) = ax^2 + bx + c$ is the starting point in its complete study, at various levels: - calculation level: factorization, sign and roots; - functional level: extremum;

CONTENTS	OBJECTIVES	COMMENTS
	4. Calculate the sum and the product of the roots of a quadratic trinomial in terms of its coefficients. 5. Write a quadratic equation knowing the sum and the product of its roots. 6. Solve a quadratic inequality with numerical coefficients. • Write a polynomial of degree two in one unknown in its standard form. • Study the existance of the real roots of a quadratic equation in one unknown (discriminant) and determine its roots when they exist. • Calculate the sum and the product of the roots of a polynomial of degree two in terms of the coefficients. • Solve problems leading to quadratic equations in one or two unknowns. • Study the sign of a polynomial of degree two. • Solve inequalities and systems of inequalities of degree two in one unknown. • Interpret graphically the solution of an equation or an inequality of degree two in one unknown. • Solve graphically a quadratic inequality.	- Graphical level : axis of symmetry and the graph deduced from that of the function $x \mapsto ax^2$. We initiate the student to handle quadratic equations with parametric coefficients. Whenever we have a special case (common identity where one of the coefficients is zero), we use it to simplify solving the equation. In case calculating the roots is complicated, finding the sum and the product of these roots may indicate their sign and allow one to find numerical values of expressions that depend only on the sum and product. The student will have to solve equations and problems leading to quadratic equations such as : finding two numbers knowing their sum and their product, biquadratic equations, finding points of intersection of two curves and finding tangents. The student will have to master reading the sign of a quadratic polynomial, the product or the quotient of two linear factors, and to use this reading in solving inequalities or systems of inequalities in one unknown.

4. POLYNOMIALS (4 h)

In the first year, the student has already seen the division of a polynomial $P(x)$ by $(x - a)$ where a is a root of $P(x)$.
He also used different methods to divide $P(x)$ by $(x - a)$ in order to solve the polynomial equation $P(x) = 0$.
This year, he will deal with division of a polynomial $A(x)$ by another $B(x)$ to find the remainder and the quotient, and to factorize and simplify rational fractions.

CONTENTS	OBJECTIVES	COMMENTS
4.1. Euclidean division of a polynomial by another.	1. Perform the Euclidean division of a polynomial by another. • Perform the division of a polynomial, having a as a root, by $(x - a)$. • Perform the Euclidean division and recognize the quotient and remainder. • Know that $P(a)$ is the remainder of the division of a polynomial $P(x)$ by $(x - a)$.	We will mention that the Euclidean division of a polynomial $A(x)$ by a polynomial $B(x)$ not identically zero, is an operation that aims to find two polynomials $Q(x)$ (quotient) and $R(x)$ (remainder) such as : $A(x) = B(x) . Q(x) + R(x)$. One should accept without proof the existence and uniqueness of the quotient and remainder. Finding them could be achieved by the method of identical polynomials or by the technique of direct division.
4.2. Factorization. Simplification of rational fractions.	1. Use factorization to simplify a rational fraction. 2. Factorize a quadratic trinomial with real coefficients. 3. Factorize a polynomial of degree three where one is either know or easy to find. • Simplify a rational fraction. • Factorize a quadratic trinomial $f(x) = ax^2 + bx + c \quad (a \neq 0, b, c \in \mathbf{R})$ in $f(x) = a(x - \alpha)(x - \beta)$, where α and β are real numbers. • Factorize a polynomial of degree three knowing one root. • Factorize a polynomial of degree three where one root is easy to find.	We note that every simplification should be performed in the domain of definition of a fraction. Having learned in the first year to find factors of the form $(x - a)$ of a polynomial of degree three, this year the student has to complete his learning.

5. NUMBERS (8 h)

Quadratic equations with real coefficients which do not have roots in **R** are analogous to certain equations of the form $x + a = b$ with coefficients in **N** which do not have solutions in **N**. To solve this problem, it is necessary to enlarge the system **R** of real numbers.

In fact, the benefit in introducing complex numbers is much beyond this necessity. Various applications in mathematics (problems involving angle or distance) and physiques (mainly electricity) justify the space that it requires.

At the level of this class, one should only make the student familiar with handling complex numbers, calculus and geometric configuration.

CONTENTS	OBJECTIVES	COMMENTS
5.1. Complex numbers : definition, algebraic form.	1. Identify a complex number and write it in the algebraic form $a + ib$. 2. Characterize a complex number equal to zero. 3. Characterize two equal complex numbers. • Verify whether a quadratic equation with real coefficients has real roots or not. • Accepting without proof the existance of a number i such that $i^2 = -1$, show that a quadratic equation with real coefficients and negative discriminant has two complex roots $\alpha + i\beta$ et $\alpha - i\beta$. • Recognize a complex number in the form $z = a + ib$, where a and b are real. • Know and use the fact that a complex $z = a + ib$ is zero if and only if $a = b = 0$. • Know and use the fact that a complex number can be written uniquely in the form : $a + ib$ $(i^2 = -1)$. • Identify the real part and the imaginary part of a complex number. • Recognize a pure imaginary complex number. • Know and use the fact that two complex numbers are equal if and only if they have the same real part and the same imaginary part.	We will consider examples showing that the set of real numbers is insufficient for solving all quadratic equations with real coefficients. We may define a complex number by various means; however whatever the method used, it is necessary to recognize that the set of complex numbers is an extension of the set of real numbers (thus every real number is a special complex number). A complex number z will be denoted by $z = a + ib$ where $i^2 = -1$. The real part a of z is denoted by $Re(z)$ and the imaginary part b is denoted by $Im(z)$. The set of complex numbers is denoted by: **C**.

CONTENTS	OBJECTIVES	COMMENTS
5.2. Operations on complex numbers.	1. Perform operations on complex numbers. 2. Solve a quadratic equation with real coefficients and a negative discriminant. 3. Calculate the conjugate of a complex number and use its properties. • Add, subtract and multiply two complex numbers. • Simplify a complex expression to the algebraic form $a + ib$. • Know and use the fact that the product of two complex numbers is zero if and only if one of them is zero. • Know the fact that a complex number has two opposite square roots and calculate them. • Solve a quadratic equation with real coefficients and a negative discriminant. • Recognize and calculate the conjugate $\bar{z}$ of a complex number. • Know and use the following properties a) $\overline{z + z'} = \bar{z} + \bar{z}'$; b) $\overline{zz'} = \bar{z}\bar{z}'$; c) $\bar{\bar{z}} = z$; d) $\overline{\left(\frac{z}{z'}\right)} = \frac{\bar{z}}{\bar{z}'}$ and $\overline{\left(\frac{1}{z}\right)} = \frac{1}{\bar{z}}$;	Operations on complex numbers will be introduced, by extensions, starting with those defined in **R**, and replacing i^2 by -1 whenever we meet i^2. This idea seems logical if we view the set of complex numbers as being the result of adjoining i to the set **R** with all the allowed rules of calculations which enable us to solve every quadratic equation with real coefficients. We denote $\bar{z}$ by the conjugate of z and notice that the complex roots of a quadratic equations with real coefficients and negative discriminant are always conjugates. Calculating the square roots of a complex number is a delicate operation. It is recommended to make the student master efficient techniques in this area and be limited to relatively simple activities. The symbol $\sqrt{\ }$ is not used to denote one of the square root of a non-real complex number by lack of justification.

CONTENTS	OBJECTIVES	COMMENTS
	e) $z\bar{z} = a^2 + b^2$ for $z = a + ib$; f) $\frac{1}{z} = \frac{\bar{z}}{z\bar{z}} = \frac{\bar{z}}{a^2+b^2}$ for $z = a + ib$; g) z is real if and only if $z = \bar{z}$; h) A complex number z is pure imaginary, if and only if $z \neq 0$ and $\bar{z} = -z$; i) $Re(z) = \frac{1}{2}(z+\bar{z})$ and $Im(z) = \frac{1}{2i}(z-\bar{z})$. • Know and use the fact that the Cartesian images of z and $\bar{z}$ are symmetric with respect to the x-axis. • Divide a complex number by another number different from zero.	
5.3. Geometric representation of a complex number..	1. Represent geometrically a complex number. 2. know the fact that the function from the set of points $p(x,y)$ in the plane to the set of complex number, which assigns to each point $p(x,y)$ the complex number $z = x + iy$, is a bijection. • Plot the image of a complex number in a plane with two orthonormal axes. • Draw the image (with *O as origin*) of a complex number and determine the endpoint of a vector with initial point *O* • Determine the set of points that satisfy a given condition.	We will exploit the uniqueness of the algebraic form of a complex number to find a bijection between the points of a plane and the elements of **C**. We underline the geometric representation of a few basic complex numbers such as 1, i, $2i$, $1 + i$, $1 - i$ and their negatives. We will make sure to emphasize the geometric configurations of a complex number and its conjugate. The activities having to do with sets of points that satisfy a given condition will be relatively simple The notions of magnitude and argument are outside of this program.

GEOMETRY (59 h)

1. CLASSICAL STUDY (18 h)

This year, the aim of the program is not only to represent physical objects by plane figures but also to apply what we learned in plane geometry to spatial situations, in order to isolate special properties of orthogonality and the computation of distances, area and volumes.

CONTENTS	OBJECTIVES	COMMENTS
1.1. Orthogonality in space.	1. Characterize the orthogonality of two lines. 2. Characterize the orthogonality of a line and a plane. 3. Characterize two perpendicular planes. 4. Relate orthogonality and parallelism. • Recognize two orthogonal lines in space. • Recognize the orthogonality of a line and a plane. • Recognize the angle of a line and a plane. • Recognize the angle of two secant planes. • Recognize two perpendicular planes. • Know and use the following properties : P_1 : If two lines are orthogonal, every line parallel to one is orthogonal to the other. P_2 : If two lines are parallel, every line orthogonal to one is orthogonal to the other. P_3 : If two lines are parallel, every plane orthogonal to one is orthogonal to the other. P_4 : If two lines are perpendicular to the same plan, they are parallel. P_5 : If two planes are parallel, every line perpendicular to one is perpendicular to the other.	The student is already familiar with plane representation of objects in space and properties of parallelism. This year we will work hard to isolate the properties of orthogonality starting with simple situations based on parallelepiped and especially the cube. Various technique of proof, especially proof by contradictions could be consolidated through proving a few of the properties P_1 , ……. , P_9 . We emphasize the generalization of notations learned in plane geometry to the corresponding notions in space geometry. Examples :

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P_6 : If two planes are perpendicular to the same line, they are parallel.
P_7 : If a plane (P) is orthogonal to a line (D) and cuts this line at a point A, then every line passing through A and orthogonal to (D) is contained in (P).
P_8 : Through a given point, there is a unique plane orthogonal to a given line.
P_9 : Through a given point, there is a unique line perpendicular to a given plane.

• Recognize the mediator plane of a segment.
• Characterize the mediator plane of $[AB]$ as the set of points equidistant from A and B.
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<table>
<tr><th>In the plane</th><th>In space</th></tr>
<tr><td>Through a point, there is a unique line perpendicular to a given line.</td><td>Through a point, there is a unique line perpendicular to a given plane.</td></tr>
<tr><td>Through a point, there is a unique line perpendicular to a given line.</td><td>Through a point, there is a unique line perpendicular to a given line.</td></tr>
<tr><td>Two lines perpendicular to a third are parallel.</td><td>Two lines perpendicular to a plane are parallel.</td></tr>
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The study of the orthogonal symmetry with respect to a plane is a desirable activity.
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<tr><td>1.2. Projections in space.</td><td>
1. Characterize the projections of a point and a plane figure on a plane parallel to a given direction.
2. Characterize the projections of a point and a vector on a line parallel to a given plane.
3. Deduce the properties of orthogonal projection on a plane and a line.

* Projection parallel to a line (D) on a plane (P).
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In this year we extend the notions of projection seen already in the plane to space.
The image A' of a point A, obtained by a projection parallel to a direction (Δ') on a line (Δ) is denoted by pr(A). Thus, pr($[AB]$) denotes the projection of the segment $[AB]$.
By applying P_1 and P_2, we deduce properties concerning invariance :
- of parallelism;
- of colinearity;
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• Determine the projections :
- of a point;
- of a line (in case this line is parallel to (D) or (P));
- of a vector $\overrightarrow{AB}$;
- of a plane figure F (in case the plane of F is parallel to (D) or (P)).
• Know and use the following properties:
P_1 : The projection of $\vec{u} + \vec{v}$ is the sum of the projections of $\vec{u}$ and $\vec{v}$;
P_2 : The projection of $k\ \vec{v}$ is the product of k and the projection of $\vec{v}$.
• Recognize the orthogonal projection on (P).
• Know and use the following property :
If $[A'B']$ is the orthogonal projection of [AB] on (P), then $A'B' = AB\ .\ \cos\alpha$ where α is the acute angle of (AB) and ($A'B'$) .
• Know and use the following property :
If s denotes the area of a plane figure F, and s' that of its orthogonal projection, then $s' = s.\cos\alpha$ where α is the acute angle of (P) and the plane of F.

* Projection parallel to a plane (P) on a line (D).

• Determine the projections :
- of a point;
- of a vector $\overrightarrow{AB}$.
</td><td>
- of the center of gravity of a triangle;
- of the equality of vectors.
The orthogonal will be used also for reference, and for calculating distances and areas.
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CONTENTS	OBJECTIVES	COMMENTS
	• Know and use, in the case of orthogonal projection on an axis $(O, \vec{i})$, the following property : $\overline{A'B'} = AB \cdot \cos(\vec{i}, \overrightarrow{AB})$ where $\overrightarrow{A'B'}$ is the projection of $\overrightarrow{AB}$ on the axis.	
1.3. Solids.	1. Recognize a prism, a pyramid, a cone, a cylinder and a sphere. 2. Know the expression of the lateral area and the volume of each of these solids. 3. Determine the intersection of a cone and a cylinder with a plane parallel to the base. 4. Study the relative position of a plane. • Know various solids: - The prism and its principal elements; - The pyramid and its principal elements; - The cone and its principal elements; - The cylinder and its principal elements; - The sphere and its principal elements. • Know and use the formulas giving the lateral areas of these solids. • Know and use the formulas giving the volumes of these solids. • Determine the intersection of a cone with a plane parallel to the base. • Determine the intersection of a cylinder with a plane parallel to the base. • Distinguish the three positions of a plane with respect to a sphere. • Determine the intersection of a sphere with a plane.	The student will use the principal elements of the following solids prism, pyramid, cone, cylinder and sphere, to calculate the lateral areas and volumes. The formulas giving the lateral areas and volumes will be assumed without proof.

2. STUDY OF VECTORS (16 h)

The study of vectors which is in the program of this year is an extension to space of the study of vectors in the plane. Vector calculus in space is a tool which contributes to the study of a few geometric properties, in preparation to computing certain magnitudes : distances, areas and volumes.

It is important that the student isolates under certain conditions, a reference of a given geometric figure, and use it to solve a given problem .

Suggested deductic material :

- tracing paper, cross-ruled paper, color pencils.
- appropriate computer and software.

CONTENTS	OBJECTIVES	COMMENTS
2.1. Vectors and reference frame in space.	1. Characterize three coplanar vectors. 2. Determine a base and a reference frame of space to locate points in space. 3. Determine vector and scalar components of a vector. 4. Determine the coordinates of a point in two systems with the same base. • Recognize geometrically three coplanar vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$. • Characterize the plane determined by three non-colinear points A,B,C as the set of points M such that : $\overrightarrow{AM} = \alpha \overrightarrow{AB} + \beta \overrightarrow{AC}$, α and β being arbitrary real constants.	The definition of a vector, the notations and properties in plane geometry extend to space. In addition, the student will learn a new notation, that of the three vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$, coplanar by an equality of the form $\vec{w} = \alpha \vec{u} + \beta \vec{v}$ where α and β are two non-zero real numbers. Note that in space with Cartesian coordinates system $(O ; \vec{i}, \vec{j}, \vec{k})$: • The line of reference $(O ; \vec{i})$ is the axis of abscissas, the line of reference $(O ; \vec{j})$ is the axis of ordinates, and the line of reference $(O ; \vec{k})$ is the axis of elevation. • The notation $A(x, y, z)$ means that the point A has as abscisse x, as ordinate y and z as elevation.

CONTENTS	OBJECTIVES	COMMENTS
	• Know and use the property: $\vec{u}$, $\vec{v}$ et $\vec{w}$ are coplanar if any only if there exist two real numbers α and β such that $\vec{w} = \alpha\,\vec{u} + \beta\,\vec{v}$ (or $\vec{v} = \alpha\,\vec{u} + \beta\,\vec{w}$ or $\vec{u} = \alpha\,\vec{v} + \beta\,\vec{w}$) . • In space, recognize a Cartesian coordinates system $(O; \vec{i}, \vec{j}, \vec{k})$ of origine O and base defined by three non-coplanar vectors $\vec{i}$, $\vec{j}$ et $\vec{k}$. • Know the fact that for evry vector $\vec{u}$ in a base $(\vec{i}, \vec{j}, \vec{k})$, there exists a unique triplet (x, y, z) of real numbers such that : $\vec{u} = x\vec{i} + y\vec{j} + z\vec{k}$. • Identify the vector and scalar (coordinates) coponants of a vector, in a Cartesian coordinates system. • Know the fact that for every M of space with Cartesian coordinates system $(O; \vec{i}, \vec{j}, \vec{k})$, there exists a unique triplet (x, y, z) of real such that $\overrightarrow{OM} = x\vec{i} + y\vec{j} + z\vec{k}$ and that x, y and z are the coordinates of M . • Plot a point $M(x, y, z)$ in a Cartesian coordinates system. • Characterize analytically the colinearity of $\vec{V}(X, Y, Z)$ and $\vec{V'}(X', Y', Z')$ by the existance of a real α such that : $X' = \alpha X$, $Y' = \alpha Y$ and $Z' = \alpha Z$.	• The notation $\vec{V}(X, Y, Z)$, $\vec{V}\begin{vmatrix} X \\ Y \\ Z \end{vmatrix}$ or $\vec{V}\begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$ means that : the first coordinate of $\vec{V}$ is X, the second coordinate is Y and the third coordinate is Z . One should note that analytic expressions of the properties of vectors in space form, in general, an extension of their analytic expressions in the plane. We would simply add a third dimension. It is desirable to avoid situations which do not follow this rule.

CONTENTS	OBJECTIVES	COMMENTS
	• Know and use the following relations : $X_{\overrightarrow{AB}} = x_B - x_A$; $Y_{\overrightarrow{AB}} = y_B - y_A$; $Z_{\overrightarrow{AB}} = z_B - z_A$. • Know the fact that the equality of two vectors $\vec{V}(X, Y, Z)$ and $\vec{V'}(X', Y', Z')$ is characterized by the equations $X = X'$, $Y = Y'$ et $Z = Z'$. Know and use the relations : $X_{\vec{U}+\vec{V}} = X_{\vec{U}} + X_{\vec{V}}$; $Y_{\vec{U}+\vec{V}} = Y_{\vec{U}} + Y_{\vec{V}}$ et $Z_{\vec{U}+\vec{V}} = Z_{\vec{U}}$ and $X_{k\vec{V}} = k.X_{\vec{V}}$; $Y_{k\vec{V}} = k.Y_{\vec{V}}$ et $Z_{k\vec{V}} = k.$ • Calculate the coordinates of a point in space defined by a vector equation (in case of the midpoint of a segment, and of the center of gravity of a triangle). • Apply analytically the colinearity of vector to prove that three points are collinear. • Know the relation between the coordinates of a point in two Cartesian coordinates system with the same base (translation of system) • Know the fact that the coordinates of a vector remain invariant in passing from a system $(O; \vec{i}, \vec{j}, \vec{k})$ to a system $(O'; \vec{i}, \vec{j}, \vec{k})$ having the same. • Know various systems : - direct (indirect); - normed; - orthogonal; - orthonormal.	

CONTENTS	OBJECTIVES	COMMENTS
2.2. Barycenter.	1. Characterize the barycenter of n weighted points. 2. Determine the coordinates of the barycenter in planar or spacial Catesian coordinates system. • Recognizre the following properties : - Si $\alpha + \beta = 0$, then the vector $\alpha\,\overrightarrow{MA} + \beta\,\overrightarrow{MB}$ is independent of M; - If $\alpha + \beta + \gamma = 0$, then the vector $\alpha\,\overrightarrow{MA} + \beta\,\overrightarrow{MB} + \gamma\,\overrightarrow{MC}$ is independent of M; - If $\alpha + \beta + \gamma + \delta = 0$, then the vector $\alpha\,\overrightarrow{MA} + \beta\,\overrightarrow{MB} + \gamma\,\overrightarrow{MC} + \delta\,\overrightarrow{MD}$ is independent of M. • Identify the barycenter G of two wheighted points. • Know and use the properties of the barycenter G of two weighted points $A(\alpha)$ and $B(\beta)$; $(\alpha + \beta \neq 0)$: - G belongs to the line (AB); - $\alpha\,\overrightarrow{MA} + \beta\,\overrightarrow{MB} = (\alpha+\beta)\,\overrightarrow{MG}$ for every point M. • Construct the barycenter G of two weighted points. • Identify the barycenter of a system of three weighted points. • Recognize and use the properties of the barycenter G of three weighted points $A(\alpha)$, $B(\beta)$ and $C(\gamma)$; $(\alpha + \beta + \gamma \neq 0)$: - G belongs to the plane (ABC); - G is the barycenter of a system consisting of one of these points, associated with its coefficients and the barycenter of the two rremaining points, aassociated with the sum of their coefficients.	$n \leq 4$. Inspite of te fact that barycentric calculus developed initially in response to the needs of physicists, it has become a very effective tool in geometric proof. One may use it to simplify vectorial sums, to prove colinearity of several points or concurence of several lines. The student has to relate the existence and the uniqueness of the barycenter of a system of weighted points to the sum of the coefficients associated with these points. He will also note that when we multiply all the coefficients by the same non-zero real number, the barycenter remains invariant. The student will benefit from partial barycenters in constructing the barycenter of three or four weighted points, performing some proofs and determining geometric locii.

CONTENT	OBJECTIVES	COMMENTS
	- $\alpha\,\overrightarrow{MA} + \beta\,\overrightarrow{MB} + \gamma\,\overrightarrow{MC} = (\alpha+\beta+\gamma)\,\overrightarrow{MG}$ for every point M. • Construct the barycenter G of three weighted points. • Identify the barycenter of four weighted points and use its properties. • Identify the isobarycenter of n points ($n \leq 4$) and characterize geometrically in case $n = 2$ and $n = 3$. • Determine the coordinates of the barycenter of a system of weighted points in a Cartesian system. • Use the partial barycenter to : construct a baycenter, show that several points are collinear and prove that several lines are concourant.	
2.3. Vector product.	1. Characterize the vector product of two vectors. 2. Know the properties of the vector product. 3. Isolate a vector normal to a plane. • Recognize vector product of two vectors $\overrightarrow{u}$ and $\overrightarrow{v}$. • Characterize the position of the point C defined by $\overrightarrow{OC} = \overrightarrow{OA} \wedge \overrightarrow{OB}$ where O, A and B are given. • Know and use the following properties : P_1 : $\overrightarrow{u} \wedge \overrightarrow{v} = -\,\overrightarrow{v} \wedge \overrightarrow{u}$; P_2 : $(\alpha\,\overrightarrow{u}) \wedge \overrightarrow{v} = \overrightarrow{u} \wedge (\alpha\,\overrightarrow{v}) = \alpha\,(\overrightarrow{u} \wedge \overrightarrow{v})$;	The properties P_1 and P_4 will be verified whereas P_2 and P_3 will be accepted. The vector product of two vectors $\vec{u}$ and $\overrightarrow{v}$ is denoted by $\overrightarrow{u} \wedge \overrightarrow{v}$ or $\overrightarrow{u} \times \overrightarrow{v}$. The student will use the vector product mainly to determine a vector normal to a plane and to characterize the colinear of two vectors, the analytic expressions of the vector product and their applications will be seen in the third year of secondary education.

CONTENTS	OBJECTIVES	COMMENTS
	P_3 : $\vec{u} \wedge (\vec{v} + \vec{w}) = (\vec{u} \wedge \vec{v}) + (\vec{u} \wedge \vec{w})$; P_4 : $\vec{u} \wedge \vec{v} = \vec{0}$, if and only if the non-zero vectors $\vec{u}$ and $\vec{v}$ are colinear. • Use the norm of $\overrightarrow{OA} \wedge \overrightarrow{OB}$ to calculate the area of the parallelogram of sides [OA] and [OB] and the distance d of the point A to (OB). • Know and use the fact that $\overrightarrow{OA} \wedge \overrightarrow{OM}$ remains invariant if M varies on a line parrallel to (OA). • Know how to isolate a vector normal to a plane define by two non-colinear vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$.	

3. ANALYTIC STUDY (9 h)

CONTENTS	OBJECTIVES	COMMENTS
3.1. Equation of a circle.	1. Characterize the Cartesian equation of a circle in an orthonormal system. 2. Relate the division of the plane of a circle into region to the power of a point with respect to a circle. 3. Determine the equation of the tangent to a circle at a given point of this circle. • Know and use the equation of the circle of center $I(a ; b)$ and of radius R : $(x-a)^2+(y-b)^2=R^2$. • Determine the center and radius of a circle knowing its equation. • Characterize the circle of diameter $[AB]$ as the set of points M such that $\overrightarrow{AM}.\overrightarrow{BM}=0$. • Know and use the aquation of a circle of diameter $[AB]$: • $(x-x_A)(x-x_B)+(y-y_A)(y-y_B)=0$. • Characterize the disc as being the set of M $(x; y)$ such that : $(x-a)^2+(y-b)^2 \leq R^2$. • Calculate the power P of a point M with respect to a circle $C(I; R)$ by the relatuon $P=\overline{MI}^2-R^2$. • Use the power of a point to determine its position with respect to a circle. • Study the relative position of a line and a circle, and determine their points of intersection if they exist.	The purpose of this chapter is to use the scolar product to translate analytically the properties of a circle and the various positions of a line with respect to a circle. This was seen already in preceding classes. The scolar product represents a tool for finding an orthonormal system $(O; \vec{i}, \vec{j})$ the equation of a circle of center I and of radius R, or the circle of diameter $[AB]$. The notion of power of a point with respect to a circle is used to determine the position of a point with respect to this circle, it also allows one to identify a quadrilateral inscribed in a circle. We should make sure to relate: - The intersection of a line and a circle to the solution of a quadratic equation; - The equation of a circle passing through three given points to the solution of a system of equations of degree one. In a circle of center I that passes through a point M, the word "radius" may denote either the segment $[IM]$ or its length. We will make the student familiar with various means to find the equation of the tangent line that passes through a point of a circle. The lines of type $\overrightarrow{MA}^2+\overrightarrow{MB}^2=k^2$ et $\frac{MA}{MB}=k$ are desirable applications.

CONTENTS	OBJECTIVES	COMMENTS
	• Determine the equation of a line tangent to a circle at a point of this circle. • Determine the equations of lines tangent to a circle through a point exterior to this circle.	
3.2. Scolar product in space.	1. Find the analytic expression of the scalar product in an orthonormal system in space. 2. Calculate the norm of a vector the distance between two points and the cosine of the angle of two vectors. In this chapter, the reference frame of space is $(O ; \vec{i}, \vec{j}, \vec{k})$. • Know and use the analytic expression $XX' + YY' + ZZ'$ *of* the scalar product of two vectors $\vec{V}(X, Y, Z)$ and $\vec{V'}(X', Y', Z')$. • Know the condition for two vectors $\vec{V}(X, Y, Z)$ and $\vec{V'}(X', Y', Z')$ to be orthogonal in the analytic form : $XX' + YY' + ZZ' = 0$. • Calculate analytically the norm $\sqrt{X^2+Y^2+Z^2}$ of a vector $\vec{V}(X, Y, Z)$. • Calculate the distance between two points $A_1(x_1, y_1, z_1)$ and $A_2(x_2, y_2, z_2)$ using the relation: $A_1A_2 = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$. • Know and use the relation $\cos(\vec{V}, \vec{V'}) = \dfrac{\vec{V}.\vec{V'}}{\Vert\vec{V}\Vert.\Vert\vec{V'}\Vert}$.	It is important to recall the definition and properties of the scolar product in the plan (first year secondary), in order to introduce the scolar product of two vectors $\vec{u}$ and $\vec{v}$ in space. Given a point A and the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ tsuch that: $\vec{u} = \overrightarrow{AB}$ and $\vec{v} = \overrightarrow{AC}$, we pass from the context of space to that of plane (ABC). The product is used in space as well as in the plane, to prove that two lines are orthogonal, to calculate distances and angles and to find same geometric loii. It is desirable to characterize a plane and a sphere respectively by $\vec{u}.\overrightarrow{AM} = 0$ et $\overrightarrow{AM}.\overrightarrow{BM} = 0$. We will make sure to deal with problems that underline the scolar product as a tool that facilitates certain proofs.

4. PLANE TRANSFORMATIONS (16 h)

In the middle cycle, the transformations were introduced experimentally, as relating an intial state to a final state of a displaced plane figure. In the the secondary cycle, a transformation is viewed as a bijective mapping in the plane. Hence, the student will notice that every point of a plane has an image, not only the points that are represented in the given figure. we will base ourselves on the properties of bijections to prove that the composition of two transformations is a transformation and that the inverse transformation is also a transformation. We will base ourselves on the fact that this year, the transformations we study are isometries in order to show the properties of invariance (collinear, parallelism, barycenters, angles, distances and areas, ...) and their consequences.

In the study of a transformation, we will deal with :

- the construction of the image of a point and a figure;
- the effect of each transformation on parallelism, barycenter, angles, distances and areas;
- in case of two isometric figures, finding isometry which transforms one to the other.

Transformations will be studied in order to be used as tools in solving problems of geometric configurations, construction and geometric locii.

CONTENTS	OBJECTIVES	COMMENTS
4.1. Isometry. Translation.	1. Characterize an isometry. 2. Characterize a translation. 3. Study the effect of a translation on geometric figures. • Recognize a point transformation in the plane . • Define an isometry (invariance of distances). • Recognize an isometry. • Recognize a translation $t_{\vec{V}}$ by a vector $\vec{V}$. • Recognize the special translation by the zero vector. • Know that the composition of a translation $t_{\vec{V}}$ followed by a translation $t_{\vec{V'}}$ is the translation $t_{\vec{V}+\vec{V'}}$.	The student already knows how to translate a figure in the plane in making it glide along a given support in a given direction by a given distance; so that a relation is established between translation and vector. We isolate the properties of a translation starting with those of bijections and vectors. We emphasize the link between vectoriel sum and composition of two translations. We denote by $t_{\vec{V}}$ the translation by a vector $\vec{V}$. One should make sure to induce the student to underline a convenient translation in geometric configurations containing key figures (parallelogram, circles of equal radius ....).

CONTENTS	OBJECTIVES	COMMENTS
	• Recognize $t_{-\vec{v}}$ the inverse translation of $t_{\vec{v}}$. • Know and use the properties of a translation : P_1 : It preserves distances (isometry); P_2 : It preserves colinearity; P_3 : It preserves parallelism; P_4 : It preserves the midpoint of a segment P_5 : It preserves the measure of oriented angles; P_6 : It preserves orthogonality; P_7 : It preserves areas; P_8 : It preserves barycenter.	.
4.2. Plane rotation.	1. Characterize a plane rotation. 2. Study the effect of a plane ratation on plane geometric figures. • Recognize the rotation $r(O, \alpha)$ of center O and angle α. • Know and use the fact that the image by $R(O, \alpha)$ of the vector $\overrightarrow{AB}$ is a vector $\overrightarrow{A'B'}$ such that $A'B' = AB$ and $(\overrightarrow{AB}, \overrightarrow{A'B'}) = \alpha + 2k\pi$, $k \in Z$. • Determine the image of a point a line, a vector and a circle, by $r(O, \alpha)$. • Know and use the following properties of rotation: P_1 : it preserves distances (isométry); P_2 : it preserves the colinearity; P_3 : it preserves the midpoint; P_4 : it preservesparallelism; P_5 : it preservesoriented angles; P_6 : it preservesorthogonality; P_7 : it preservesareas; P_8 : it preserve barycentre.	In an oriented plane, the student should master trigonometric knowledge relative to oriented angles in order to grasp rotations It is important to note that the center of a ratation (invariant point) in not necessarily a point of the figure in question. We denote by $r(O, \alpha)$ the rotation of center O, and angle α. One should make sure to intiate the student to underline convenient rotation in geometric configurations containing key figures (equilateral triangle, square, circle of equal radius) and to use them to solve problems in geometry. We isolate the properties of a rotation starting with those of bijections and oriented angles. We emphasize the link between the sum of oriented angles and the composition of two rotations of the same center.

CONTENTS	OBJECTIVES	COMMENTS
.	• Recognize $r(O, -\alpha)$ as the inverse of $r(O, \alpha)$. • Know that the composition of a rotation $r(O, \alpha)$ followed by a rotation $r(O, \alpha')$, having the same center O, is the rotation $r(O, \alpha + \alpha')$.	We treat the central symmetry as a rotation of angle π . the composition of two rotations with distinct centers is beyond the scope of this program.
4.3. Reflexion.	1. Characterize a reflexion . 2. Study the effect of a reflection on plane geometric figures. • Recognize a reflexion s_D of axis (D). • Determine the image of a point, a line, a segment, and a circle, by s_D. • Know and apply the following properties of reflexion : P_1 : it preserves distances (isométry); P_2 : it preserves parallelism; P_3 : it preserves the midpoint of a segment; P_4 : it preserves colinearity; P_5 : it preserves the geometric angles; P_6 : it preserves orthogonality; P_7 : it preserves areas; P_8 : it preserves barycenter. • Know that the composition of a reflection with itself leaves every point in the plane invariant (involution). • Recognize and construct the axis of symmetry of some figures. • Recognize the composition of two reflexions of axes (D) and (D') : • The case where (D) is parallel to (D'); • The case where (D) and (D') are concurent.	We would use the term "axial symmetry" or "orthogonal" instead of "reflexion". Through appropriate activities, the student will be trained to isolate the properties of a reflexion and to underline, for example, that the image of the "left" hand is the right "hand". It is important to note that, contrary to the case of translations and rotations, the composition of two reflexions is not a reflexion because the reflexion is an involution. In terms of activities, we may apply seperately a central symmeetry and a reflection to the same figure, in order to study their effects on oriented angles. We denote by : - s_D lthe reflexion of axis (D); - $s_D(A)$ the symmetric of A with respect to (D). We will link the axis of symmetry (D) of a figure (F) and the reflexion s_D, by noting that the image of (F) by s_D is invariant, in particular in the following cases: diameter of a circle, bissector of an angle, perpendicular bissector of a segment...

ANALYSYS (NUMERICAL FUNCTIONS) (42 h)

1. DEFINITIONS AND REPRESENTATION (14h)

In this class, analysis is focused mainly on the study of functions that are essentialy simple rational and irrational.
The use of a graphical calculator is desirable in class to control graphing. If available, the use of appropriate software is desirable. Likewise , an intuitive approach to limits is also desirable. As all functions studied this year are continuous on their domain of definition, it is preferable to emphazise the graphical meaning of continuity. The notion of continuous extension will be given in higher classes.
It is good to underline the practical meaning of the derrivative in geometry, kinematic and economics.
For numerical sequences we should make the students familiar with simple situations. Exhaustive treatment of sequences is to be excluded. The limit of a sequence will be studied subsequently.
The calculus of antiderrivatives will be studied only as the inverse operation of differentiation.

CONTENTS	OBJECTIVES	COMMENTS
1.1. Limit of a function. Asymptotes.	1. Identify the limit of a function at a point a of at infinity. 2. Know the limits of the basic functions. 3. Know whether a function has a vertical, horizental or oblique asymptote. 4. State and use the properties of limits. 5. Recognize an indeterminate form and remove the indetermination. • Know the fact that $\lim_{x\to a} f(x)$ has no meaning except when f is defined on an interval containing a or having a as an endpoint. • For the basic functions, know the fact that $\lim_{x\to a} f(x) = f(a)$ where a is in their domain of definition.	Using graphical examples, we show how a function tends to a limit L when x tends to a point a. Finding oblique asymptotes will be limited to rational functions. We will use the form $f(x)= L + \varphi(x)$ with $\lim_{x\to a} \varphi(x) = 0$ where a denotes one of the symbols $+\infty$ or $-\infty$ to inogurate the study of asymptotes. We emphasize the role of vertical asymptotes to distinguish between $\lim_{\substack{x\to a\\x>a}} f(x)$ and $\lim_{\substack{x\to a\\x<a}} f(x)$. Using numerical examples, we will note that if $f(x) > g(x)$, then $\lim_{x\to a} f(x) \geq \lim_{x\to a} g(x)$.

CONTENTS	OBJECTIVES	COMMENTS
	• Calculate $\lim_{x\to a} f(x)$ in simple cases. • Know the equivalences of the following writtings: - $\lim_{x\to a} f(x) = L$ - $\lim_{x\to a}(f(x) - L) = 0$ $f(x)=L+\varphi(x)$ with $\lim_{x\to a} \varphi(x) = 0$ where a denotes one of the symbols $+\infty$ ou $-\infty$. • Know that $\lim_{x\to a} f(x)$ does not necessarely exist. • Calculate $\lim_{x\to a} f(x)$ when a is an endpoint of the domain of definition of f. • Interpret geometrically in terms of asymptotes $\lim_{x\to a} f(x) = +\infty$ an $\lim_{x\to a} f(x) = -\infty$. • Calculate $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ in simple cases. • Interpret in terms of asymptotes $\lim_{x\to+\infty} f(x) = L$ and $\lim_{x\to-\infty} f(x) = L$ where L is a real number. • Interpret geometrically in terms of asymptote $\lim_{x\to+\infty} (f(x) - (ax+b)) = 0$ and $\lim_{x\to-\infty} (f(x) - (ax+b)) = 0$. • Know the behaviour of a polynomial function or a rational fraction in the neighbohood of infinity. • Determine the equation of the oblique asymptote in the case of a rational function.	We note the existance of functions having no limits at $+\infty$ or $-\infty$. The indeterminate forms that we consider are all dealt with by factorization or simplification. We will show geometrically that $\lim_{x\to 0} \frac{\sin x}{x} = 1$, where the measure of x is expressed in radians.

CONTENTS	OBJECTIVES	COMMENTS
	• Know and use the limit of a sum, a product and a quotient of two functions. • Recognize the indeterminate forms and remove the indetermination. • Note that if $f(x) \geq g(x)$ on an interval containing a or having a as an endpoint, then $\lim_{x\to a} f(x) \geq \lim_{x\to a} g(x)$. • Note that if $u(x) \leq f(x) \leq v(x)$ on an interval containing a or having a as an endpoint and if $\lim_{x\to a} u(x) = \lim_{x\to a} v(x) = L$, then $\lim_{x\to a} f(x) = L$. • Note that if $g(x) \leq f(x)$ and a is an endpoint of the domain of difinition of f *and* g, then: $\lim_{x\to a} g(x) = +\infty \Rightarrow \lim_{x\to a} f(x) = +\infty$ and $\lim_{x\to a} f(x) = -\infty \Rightarrow \lim_{x\to a} g(x) = -\infty$.	
1.2. Numerical sequences. Arithmetic sequences. Geometric sequences.	1. Identify a sequence of real terms defined by the general term or by a realtion of recursion. 2. State the principal of recursion and use it to find the general term of a sequence defined by a relation of recursion of order one. 3. Caracterize an increasing or decreasing sequence. 4. Caracterize an arithmetic sequence by its first term and its ratio. 5. Calculate the general term of an arithmetic sequence and the sum of the first n terms.	A sequence of general term U_n is denoted by (U_n). A recursive sequence (U_n) is given by $\begin{cases} \text{its first term} \\ U_{n+1} = f(U_n). \end{cases}$ One should note that given U_o and a relation $U_{n+1} = f(U_n)$ does not always allow one to define a sequence (U_n). The convergence of sequences is beyond the scope of the program of this class. We will point out to the student the importance of the initial condition in reasonning by induction (a property could be hereditary without being true).

CONTENTS	OBJECTIVES	COMMENTS
	6. Characterize a grometric sequence by its firs term and its ratio. 7. Calculate the general term of a geometric sequence and the sum of the first n terms. • Recognize a numerical sequence as being a maping from a subset of **N** to **R**. • Calculate the terms of a numerical sequence. • Know and use the principal of reasoning by induction. • Study the variation of a numerical sequence by: - the sign of $U_{n+1} - U_n$; - the comparison of $\frac{U_{n+1}}{U_n}$ to 1 if the terms of the sequence are strictly positive; - the variation of the function of f if f is defined on $[0,+\infty[$ and $U_n = f(n)$. • Recognize an arithmetic sequence by its first term and its ratio. • Calculate the general term of an arithmetic sequence. • Calculate the sum of the first n terms of an arithmetic sequence. • Recognize a geometric sequence by its first term and its ratio. • Calculate the general term of a geometric sequence. • Calculate the sum of the first n terms of a geometric sequence.	We note that computing the first terms of a sequence does not allow one to deduce the global behavior of a sequence; on the other hand, it allows one to conjecture. the formulas of arithmetic and geometric sequences will serve as models for learning reasoning by induction. It is important to initiate the student to specific techniques of sequences by applying them to questions already approached with functions: increasing, decreasing, etc.

2. CONTINUITY AND DIFFERENTIATION (22h)

CONTENTS	OBJECTIVES	COMMENTS
2.1. Continuity.	1. Define the continuity of a function at a point. 2. Recognize a continuous function on a given interval. 3. Determine the intervals of continuity of the basic functions. • Know that a function f defined on an interval containing the number a is continuous at a if $\lim_{x \to a} f(x) = f(a)$. • Recognize graphically a continuous function on an interval contained in its domain of definition.. • Know that all basic functions are continuous on every interval of their domain of definition.	The notions of left and right continuity are not part of this program. Since it is difficult to understand the notion of continuity analytically it will only be approached graphically this year. We accept without proof the continuity of functions that we study on their domain of definition.
2.2. Derivative of function at a point.	1. Define the derrivative of a function at a point and give it a geometric and kinematics interpretation. • Recognize the rate of change of f at a : $\frac{f(a+h)-f(a)}{h}$, and interpret its sign. • Know that the derrivative of f *at* a is the number $A = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h} = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ when this limit exists.	In order to introduce the notion of the value of the derrivative at a point, we require from the student to calculate limits of the rate of variations of simple functions.the notion of differentiation at a point has 3 inseperable aspects: • The geometric aspect, which leads to the notion of tangent. • The numerical aspest, which introduces the approximation of a numerical function in the neiborhood of a point by a linear function. • The kinematic aspect related to the concept of instantaneous velocity.
	• Know that the volue of the derrivative of f at a is the slope of the tangent to the graph of f at the point $(a, f(a))$ and that the equation of the tangent at this point is : $y - f(a) = A(x - a)$. • Know that the instanteneous velocity at time t_0 of a moving body M, where the law of time is given by $t \mapsto f(t)$, is the derrivative of f at t_0. • Know that, if the limit of the rate of change of f at a is infinite, the tangent to the graph at the point $(a, f(a))$ is parallel to the y-axis. • Know that if the derrivative of f at a is zero, the tangent to the graph at the point $(a, f(a))$ is parallel to the x-axis.	We emphasize the geometric aspect and we will deal with the two other aspects by direct applications. We will make sure to study the relative position of a curve and its tangent at a point.
2.3. Derivative function.	1. Calculate the derrivative function of each of the basic functions. 2. State and use the theorems of differentiation. • Recognize a function differentiable on an interval. • Calculate the derrivative of the basic functions. • Know and use the derrivative of $(u + v)$, $(u \cdot v)$, (au), $\frac{u}{v}$, $\frac{1}{v}$, u^n, where u and v are differentiable functions. • Know that the domain of definition f' is a susbset of that of f but they are not always equal.	Starting with appropriate activities, we isolate the formulas of differentiations that the student should memorize. The derrivative function is mainly used in the study of functions. for this task, we make sure not to choose complicated functions. Although the study of differentiation is not an objective in itself, it is desirable for increasing the number of exercises in order to enable the student to master this calculus completely.

CONTENTS	OBJECTIVES	COMMENTS
	• Know and use the fact that a differentiable function on an interval is increasing (resp. decreasing) on this interval, if the derrivative function is positive (resp. negative); and that the derrivative of a constant function is zero. • Know that if f' is equal to zero at a point a where f changes its sign, then *f(a)* is a local extremum of *f*. • Recognize graphically a function continuous at a point but not differentiable there.	
2.4. Study of functions. Polynomial functions, rational functions.	1. Study and represent graphically a rational function and an irrational function of the form $x \mapsto \sqrt{ax+b}$. • Find the domain of definition of a function. • Reduce if possible, the domain of study by considerations of parity (add even function). • Verify that a given point is a center of symmetry of the graph of a function and that a line parallel to the *y*-axis is an axis of symmetry of this curve. • Study the limits at the endpoints of the open intervals of the domain of difinition to find the asymptotes.	Although the study of functions, in the program of this year, appears as an objective in itself, one should not forget that this study is necessary and efficient in the approximate solutions of equations, optimisation problems and comparison of functions. Thus, it is desirable that the problems should not be limited simply to the study of a given function, but that they should be extended to situations taken from other disciplines mainly geometry. If a programmable calculator is available, we should make use of it to familiarize the student with finding an approximate solution of an equation of the form *f(x)* = 0 by the method of dichotomy and scanning. We will limit ourselves to rational functions of the form $\frac{P(x)}{Q(x)}$ where $\deg(P(x)) - \deg(Q(x)) \leq 1$.

CONTENTS	OBJECTIVES	COMMENTS
	• Find the oblique asymptote of a rational function or verify that a given line is an asymptote. • Study the position of a curve with respect to its asymptote. • Syudy the position of a curve (*C*) representing a function f with respect to the tangent at a point of (*C*). • Find the derrivative and determine its sign. • Draw the table of variation summarizing the study of the function. • Draw the graph of the function.	For a good understanding of the variation of a function it would be good to graph the functions f and f' on the same graph and to read the variations of f *in terms of the graph of* f'. The study of functions such as $x \mapsto \sqrt{x}$ and $x \mapsto \lvert x^2 - 1 \rvert$ enables one to introduce the notion of vertical tangent.

3. INTEGRATION (6h)

CONTENTS	OBJECTIVES	COMMENTS
3.1. Antiderrivatives of a function continuous on an interval.	1. Identify the passing of a function to an antiderrivative as the inverse operation of differentiation. 2. Know that a constant function is an antiderrivative of the zero function, and deduce the relation between two antiderrivatives of the same function on an interval *I*. 3. List the antiderrivatives of the basic functions and verify each one. 4. Use linearity in calculating antiderivatives.	We use $\int f(x)dx$ to denote an antiderrivative defined up to a constant of the function *f*. The student will learn how to find the antiderrivative that satisfies a given condition. We will compute antiderrivatives for simple functions obtained by linear combination of the basic and simple trigonometric functions. Antiderrivatives of rational functions are not in the program.

CONTENTS	OBJECTIVES	COMMENTS
	• Know the definition of an antiderivative of a continuous function on an interval I. • Know that a constant function is an antiderrivative of the zero function. • Know that, on an interval I, two antiderrivatives of the same function differ by a constant. • Know that a function continuous on an interval I has infinitely many antiderrivatives. • Know the antiderrivatives of functions f defined on an interval I by the following expressions: x^n $(n \neq -1)$; $\frac{1}{\sqrt{x}}$; $\sqrt{x}$; $cosx$; $sinx$; $\frac{1}{cos^2 x}$; $\frac{1}{sin^2 x}$; $cosax$; $sinax$ $(a \neq 0)$ • Find an antiderrivative of a function satisfying a given condition. • Find an antiderrivative of a function by decomposing it into sum of functions whose antiderrivative s are known • Know that kF is an antiderrivative of kf where F is an antiderrivative of f *and* k is a constant. • Linearize a trigonometric polynomial in order to calculate its antiderrivative.	We accept without proof that every continuous function on an interval I has an antiderrivative on I.

TRIGONOMETRY (15 h)

1. TRIGONOMETRIC LINES (4 h)

The notion of oriented angle, seemingly of little importance in classical geometry, finds its field of application in trigonometry and in the study of transformation; thus we will talk about transformations that preserve oriented angles and transformations that do not preserve them.

CONTENTS	OBJECTIVES	COMMENTS
1.1. Oriented angle of two vectors	1. Define the angle of two unit vectors, of two arbitrary non-zero vectors. Principle measure. Zero angle. 2. Add two angles and use chasles relation. 3. Measure the oriented angle of two vectors. 4. Define the polar coordinates of a point in the plane with respect to a polar axis $(O,\vec{i})$. • Recognize the angle of two unit vectors. • Recognize the angle of two vectors. • Calculate the principle measure of the angle of two vectors . • Know chasles relation relative to oriented angles • Know that : $(\vec{v}, \vec{u}) = -(\vec{u}, \vec{v}) + 2k\pi$ $(-\vec{u}, \vec{v}) = (\vec{u}, \vec{v}) + \pi + 2k\pi$ $(-\vec{u}, \vec{u}) = \pi + 2k\pi$ $(\vec{u}, \vec{v}) = (-\vec{u}, -\vec{v}) + 2k\pi = (\alpha\vec{u}, \alpha\vec{v}) + 2k'\pi$; $\alpha \in R^*$ $(\vec{u}, \vec{u}) = 0 + 2k\pi$.	Passing from calculation of geometric angles to calculation of oriented angles presents same difficulties for the student. However, the introduction of the oriented angle of two vectors starting with that of two unit vectors and the knowledge of oriented arcs that the student has will make his work easier. We note that the notion of oriented angles of two vectors enables us to reinforce the properties of rotations. We denote by $(\vec{u}, \vec{v})$ the oriented angle of two vectors $\vec{u}$ and $\vec{v}$ as well as the measure of this angle. We say that a triangle ABC is direct if the principle measure of the angle $(\overrightarrow{AB}, \overrightarrow{AC})$ is strictly positive.

CONTENTS	OBJECTIVES	COMMENTS
	• Know that, if A, B and C are three distinct points then : $(\overrightarrow{AB}, \overrightarrow{AC}) + (\overrightarrow{BC}, \overrightarrow{BA}) + (\overrightarrow{CA}, \overrightarrow{CB}) = \pi + 2k\pi$. • Recognize a direct base. • Know the formulas of passage from polar coordinates of axis $(O,\vec{i})$ to Cartesian coordinates in a direct orthonormal system $(O;\ \vec{i},\vec{j})$, and vice versa.	
1.2. Basic trigonometric formulas.	1. Know and use the basic trigonometric formulas. • Know the formulas of addition expressing : $cos\ (a - b)$, $cos\ (a + b)$, $sin\ (a - b)$, $sin\ (a + b)$, $sin\ 2a$, $cos\ 2a$. • Calculate $cos^2\ a$ and $sin^2\ a$ in terms of $cos\ 2a$. • Know and use the formulas expressing $tan\ (a + b)$, $tan\ (a - b)$, $tan\ 2a$ in terms of $tan\ a$ and $tan\ b$. • Know and use the formulas expressing $sina$, $cosa$ and $tan\ a$ in terms of $tan \frac{a}{2}$. • Know and use the formulas of transformation of : $sin\ p + sin\ q$, $sin\ p - sin\ q$, $cos\ p + cos\ q$ et $cos\ p - cos\ q$.	The formulas of addition, of duplication of arc, of linearization and transformation complete the first notions of trigonometry studied in the first year of secondary education. To facilitate memorization, it is desirable that the student derive all the formulas from only one. Hence, he can derive formulas concerning arcs associated with a given arc. Although the metric relations in a triangle are in the program of the following year, it is good to practice geometric activities that show the usefulness and the efficiency of trigonometric formulas starting this year.

2. TRIGONOMETRIC EQUATIONS (7 h)

CONTENTS	OBJECTIVES	COMMENTS
2.1. Solution of equations of the form $sin\ x = a$, $cos\ x = a$, $tan\ x = a$.	1. Solve and discuss these equations. • Know that $sin\ x = a$ and $cos\ x = a$ have no solutions except when $-1 \le a \le +1$. • Solve the equations of the form $sin\ x = sin\ \alpha$ and $cos\ x = cos\ \alpha$. • Solve equations of the form $sin\ x = a$ and $cos\ x = a$ for a special real number a such that $\lvert a \rvert \in \{0, \frac{1}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}, 1\}$. • Use a calculator to find an approximate solution of an equation of the form $sin\ x = a$ and $cos\ x = a$ for an arbitrary real number a and complete the solution in **R** or in a given interval. • Use the trigonometric circle to solve the equation $sin\ x = a$ and $cos\ x = a$ where a is real. • Solve the equation of the form $tan\ x = tan\ \alpha$. • Solve the equation of the form $tan\ x = a$ where $\lvert a \rvert \in \{0, \frac{\sqrt{3}}{3}, \sqrt{3}, 1\}$. • Use a calculator to find an approximate solution of the equation $tan\ x = a$ and complete the solution in **R** or a given interval. • Use the trigonometric circle to solve the equation $tan\ x = a$.	

3. CIRCULAR FUNCTIONS (4 h)

CONTENTS	OBJECTIVES	COMMENTS
3.1. Study of circular functions.	1. Underline the periodicity and the parity of circular functions. 2. Study the continuity and differentiability of the circular function. 3. Study the circular functions and represent them graphically. • Know that the *sine* and *cosine* functions are defined, continuous and differentiable throughout **R**. • Know that the *sine* and *cosine* function are periodic of period 2π. • Recognize the parity of the *sine* and *cosine* functions . • Know and use the derivative functions of *sine* and *cosine*. • Know that the function *cosine* is decreasing on $[0; \pi]$. • Know that the *sine function is increasing on* $[0; \pi/2]$ and decreasing on $\left[\frac{\pi}{2}, \pi\right]$. • Represent graphically the *sine* and *cosine* functions. • Know that the *tangent* function is defined, continuous and differentiable for every real x different from $(2k+1)\frac{\pi}{2}$, $k \in Z$. • Know that the *tangent* function is periodic of period π. • Know that the *tangent* function is odd.	The circular functions should not be studied outside the general context of functions especially because their study furnished the student with a large number of examples of simple functions on parity and periodicity. This study enables one to refine the solution of simple trigonometric equations.

CONTENTS	OBJECTIVES	COMMENTS
	• Know and use the derivative function of the tangent function. • Know that the tangent function is increasing on $\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$, and that $\lim\limits_{\substack{x \to -\frac{\pi}{2} \\ x > -\frac{\pi}{2}}} \tan x = -\infty$ and that $\lim\limits_{\substack{x \to \frac{\pi}{2} \\ x < \frac{\pi}{2}}} \tan x = +\infty$, and that it has as asymptotes the two lines: $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$. • Represent graphically the tangent function on $]-\frac{\pi}{2};+\frac{\pi}{2}[$. • Know that the *tangent* function is decreasing on$]0, \pi[$, $\lim\limits_{\substack{x \to 0 \\ x > 0}} \cot x = +\infty$, $\lim\limits_{\substack{x \to \pi \\ x < \pi}} \cot x = -\infty$, and that it has the as asymptotes the lines $x = 0$ and $x = \pi$.	

STATISTICS AND PROBABILITY (20 h)

1. STATISTICS (8 h)

In this class, one should work with statistical data in a continuous variable
Since statistical data in a discrete variable have already been dealt with in the first secondary, the student should now be encouraged to master the passing from a discrete variable to a continuous variable.

We point out that grouping into classes or intervals leads to a loss of information. On the other hand, various groupings for the same statistical data give a clearer idea about the study at hand.
One should note that graphical representations (histograms and polygons) do not suffice for explaining everything, they enable one however to clarify certain aspects of the study at hand.
For motivating students, it is desirable that the proposed examples be authentic and closely linked with scientific, economic and social areas.
The use of a calculator is recommended.

CONTENTS	OBJECTIVES	COMMENTS
1.1. Continuous variable, grouping into classes.	1. For the same statistical data, propose various groupings, better adapted to the study at hand. • Determine an interval $[a, b]$ of **R** that contains all the values taken by the random variable. • Recognize a class and determine its center. • Choose a partition of $[a, b]$ into a finite number of intervals (classes) with equal width. • For the same data, perform various grouping into classes. • Pass from a quantitative discrete random variable to a quantitative continuous random variable by grouping into classes.	We will limit ourselves to classes with equal width We assume that in each class or interval, the items are distributed regularly. The limits of classes should be simple values (not fractional). The number of classes to adopt depends on phenomenon at hand, on the precision of the measure desired and on the size of population at hand. When the first and last class are not determined precisely, we assume they are of the same width as other classes.

CONTENTS	OBJECTIVES	COMMENTS
1.2. Statistical data of items, frequencies; histogram, polygons.	1. Represent items and frequencies by histograms and polygons. • Translate information in a table of items and frequencies. • Represent items and frequencies by a histogram and a polygon. • Read the graph of items.	Graphical representation should be done in the plane in Cartesian coordinates and with a vertical scale called arithmetic. It should be clear and simple to visualize rapidly the general shape of the phenomenon at hand. It may serve to complete and translate a table of items and frequencies. It lends itself to comparisons with similar phenomena. We must avoid a complicated graph overstaffed with information.
1.3. Statistical data of items and cumulative frequencies; histogram, polygons.	1. Calculate the items and cumulative frequencies and represent them graphically. • Draw the table of cumulative items. • Draw the table of cumulative frequencies. • Represent cumulative items and frequencies by a histogram and a polygon. • Read a graph of cumulative items.	The curve of cumulative frequencies will be represented on the same graph as the histogram in the only case where the axes can be marked (equal width of classes).

2. PROBABILITY (12 h)

The notion of probability should be introduced intuitively. We will avoid theoretical explanation. One should train students to describe simple random experiments.
The aim and the purpose of the calculus of probabilities is to predict and calculate results of random situations, which occur continuously in daily life.
In our days, the calculus of probabilities is used in various domains: polls, insurance, meteorology, biology, physics, etc.
It is desirable to link probabilities to statistics by relating frequency to probability.
The proposed situations should be simple and should not involve combinatory difficulties.
It is recommended to use a calculator.

CONTENTS	OBJECTIVES	COMMENTS
2.1. Notion of probability.	Estimate the value of the probability of an event and verify experimentally this estimation. • Know how to estimate the value of the probability of a given situation. • Verify experimentally this estimation.	We should initiate the student to describe some random experiments of daily life using a table, or a tree to estimate the value of the probability. An event should be defined with precision, its realization should not involve any ambiguity.
2.2. Sample space. The case of equally likely events.	1. Define the terms: possibility, event, sample space, certain event, impossible event, equally likely events. • Recognize a possibility. • Recognize an event, an elementary event. • Recognize the sample space Ω. • Recognize a certain event, an impossible event $\varnothing$. • Recognize equally likely events.	We denote the certain event by Ω and the impossible event by $\varnothing$.
2.3. Properties of probability.	1. Calculate the probability of an event using the basic properties of probability. • Recognize the probability of the certain event as equal to 1 $(P(\Omega)=1)$. • Know that if $A \neq \varnothing$ then $P(A) > 0$. • Know that if A is the impossible event, then $P(A) = 0$. • Know that if $A = \{a_1, a_2, \ldots, a_n\}$, then $P(A) = P(a_1) + P(a_2) + \ldots + P(a_n)$ • Know that for an event A, $0 \leq P(A) \leq 1$.	We note that, for an event A, the formula $P(A) = \dfrac{\text{number of favorable cases}}{\text{number of possible cases}}$ is not true except in case of equally likely events.
2.4. Calculus of probabilities : event (A and B), event (A or B), disjoint events, complementary events..	1. Distinguish between these events and perform calcululation. • Recognize the event (A and B). • Recognize the event (A or B). • Recognize two disjoint events. • Recognize two complementary events. • Know that if A and B are incompatible, then $P(A \text{ and } B) = 0$ and $P(A \text{ or } B)=P(A)+P(B)$. • know that for two arbitrary events A and B, $P(A \text{ or } B)= P(A) +P(B)- P(A \text{ and } B)$. • Know that if A and $\overline{A}$ are two complementary events, then: $P(A)+P(\overline{A})=1$.	We recall previous formulas and extend them by: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A) + P(\overline{A}) = 1$ which are true only if A and B are events of the same sample space Ω. We use the formulas of arrangements and permutations.